BASIC PROPERTIES OF A PERIODICAL EIGENVALUE PROBLEM FOR A FUNCTIONAL–DIFFERENTIAL EQUATION

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The periodical boundary value problem

$$-(pu')' + qu - \int_0^l (u(y) - u(x))d_y r(x,y) = \lambda \rho u, \tag{1}$$

$$u(0) = u(l), \qquad u'(0) = u'(l). \tag{2}$$

is considered. The functions p, q, r are assumed to satisfy certain positivity and symmetry conditions. It is shown that this problem has a mechanical interpretation.

Consider the bilinear form

$$[u,v] \doteq \int_0^l (pu'v' + quv)dx + \frac{1}{2}\int_{I\times I} (u(y) - u(x))(v(y) - v(x))\, d\xi. \tag{3}$$

Using this form can be introduced a Hilbert space W with the inner product $[u, v]$. The main obtained result is the existence of a system of eigenfunctions of problem (1), (2) that forms a basis in W:

T h e o r e m. *The boundary value problem* (1), (2) *has a system of nontrivial solutions* $u_n(x)$ *corresponding to positive eigenvalues* λ_n*. This system forms an orthogonal basis in the space* W*.*

It is possible to imagine a mechanical system and to describe its vibrations as a decomposition in a series of natural vibrations. These natural vibrations can be represented as eigenfunctions of problem (1), (2).

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